CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

October/November 2003

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

[Turn over

1 Find the coordinates of the points of intersection of the line $y + 2x = 11$ and the curve $xy = 12$. [4]

2 **(i)** Show that the equation $4\sin^4\theta + 5 = 7\cos^2\theta$ may be written in the form $4x^2 + 7x - 2 = 0$, where $x = \sin^2\theta$. [1]

(ii) Hence solve the equation $4\sin^4\theta + 5 = 7\cos^2\theta$, for $0° \leqslant \theta \leqslant 360°$. [4]

3 **(a)** A debt of \$3726 is repaid by weekly payments which are in arithmetic progression. The first payment is \$60 and the debt is fully repaid after 48 weeks. Find the third payment. [3]

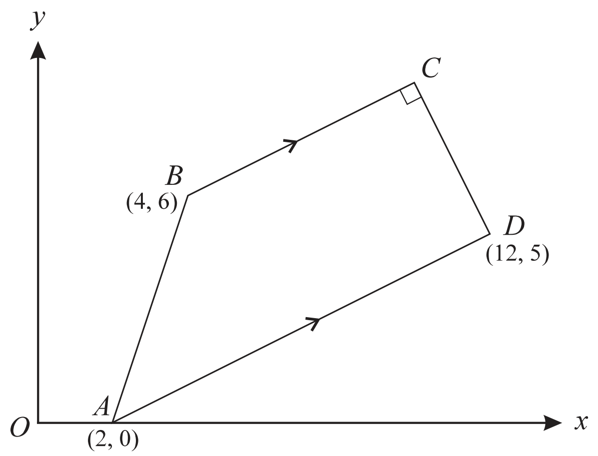
(b) Find the sum to infinity of the geometric progression whose first term is 6 and whose second term is 4. [3]

4 A curve is such that $\dfrac{dy}{dx} = 3x^2 - 4x + 1$. The curve passes through the point $(1, 5)$.

(i) Find the equation of the curve. [3]

(ii) Find the set of values of x for which the gradient of the curve is positive. [3]

5

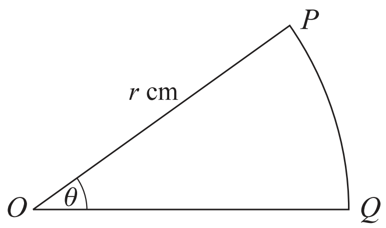


The diagram shows a trapezium $ABCD$ in which BC is parallel to AD and angle $BCD = 90°$. The coordinates of A, B and D are $(2, 0)$, $(4, 6)$ and $(12, 5)$ respectively.

(i) Find the equations of BC and CD. [5]

(ii) Calculate the coordinates of C. [2]

6



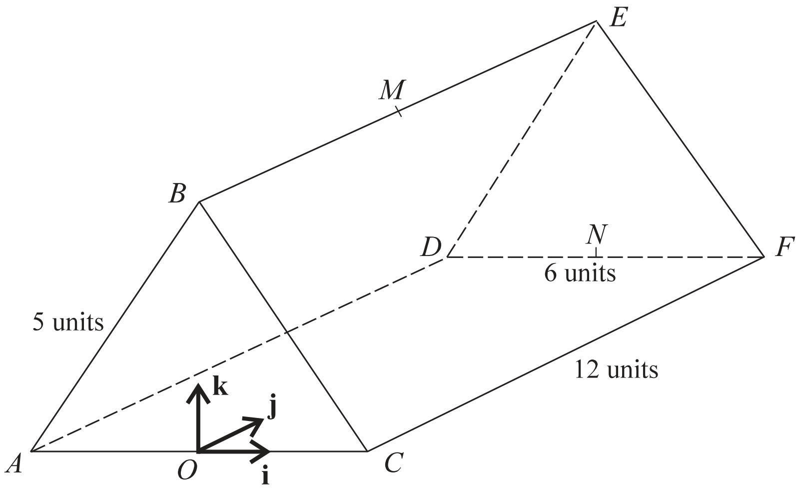
The diagram shows the sector OPQ of a circle with centre O and radius r cm. The angle POQ is θ radians and the perimeter of the sector is 20 cm.

(i) Show that $\theta = \frac{20}{r} - 2$. [2]

(ii) Hence express the area of the sector in terms of r. [2]

(iii) In the case where $r = 8$, find the length of the chord PQ. [3]

7



The diagram shows a triangular prism with a horizontal rectangular base $ADFC$, where $CF = 12$ units and $DF = 6$ units. The vertical ends ABC and DEF are isosceles triangles with $AB = BC = 5$ units. The mid-points of BE and DF are M and N respectively. The origin O is at the mid-point of AC.

Unit vectors **i**, **j** and **k** are parallel to OC, ON and OB respectively.

(i) Find the length of OB. [1]

(ii) Express each of the vectors $\overrightarrow{MC}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

(iii) Evaluate $\overrightarrow{MC}.\overrightarrow{MN}$ and hence find angle CMN, giving your answer correct to the nearest degree. [4]

8 A solid rectangular block has a base which measures $2x$ cm by x cm. The height of the block is y cm and the volume of the block is $72\,\text{cm}^3$.

(i) Express y in terms of x and show that the total surface area, $A\,\text{cm}^2$, of the block is given by

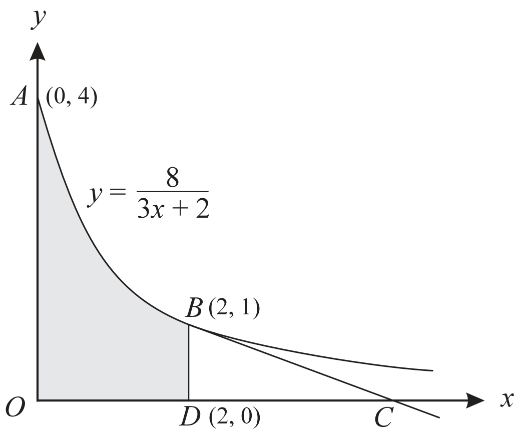
$$A = 4x^2 + \frac{216}{x}.$$ [3]

Given that x can vary,

(ii) find the value of x for which A has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

9



The diagram shows points $A\ (0, 4)$ and $B\ (2, 1)$ on the curve $y = \frac{8}{3x+2}$. The tangent to the curve at B crosses the x-axis at C. The point D has coordinates $(2, 0)$.

(i) Find the equation of the tangent to the curve at B and hence show that the area of triangle BDC is $\frac{4}{3}$. [6]

(ii) Show that the volume of the solid formed when the shaded region $ODBA$ is rotated completely about the x-axis is 8π. [5]

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x - 5, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto \frac{4}{2-x}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

(i) Find the value of x for which $\mathrm{fg}(x) = 7$. [3]

(ii) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [3]

(iii) Show that the equation $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$ has no real roots. [3]

(iv) Sketch, on a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between these two graphs. [3]